La mer 24: 3-10, 1986
Société franco-japonaise d'océanographie, Tokyo

Frequency response of fresh water content in shelf waters*

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Abstract: The frequency response characteristics of the variation of fresh water content in shelf waters to that of river discharge on the shelf is estimated by data analysis in the Bungo Channel, Huga-Nada, Tosa Bay and the Kii Channel in the south western part of Japan. The river discharge has a prominent variability with a period of one year. The amplitude ratios of the seasonal variations of fresh water content to those of river discharge change from 0.5 month to 0.9 month and phase differences of the seasonal variation of fresh water content to those of river discharge change from 0.3 month to 6 months in those areas.

1. Introduction

In recent years human activity spreads its industrial field from the coastal area to the shelf sea area as the coastal area has already no sufficient space. It is urgent to clear the characteristic variabilily of oceanic condition on the shelf sea for the harmonic development with human activity and nature there. The input of fresh water into continental shelf waters alters the physics, chemistry, biology and sediments in the shelf waters. The runoff waters are also the main source of pollution in shelf waters. How-

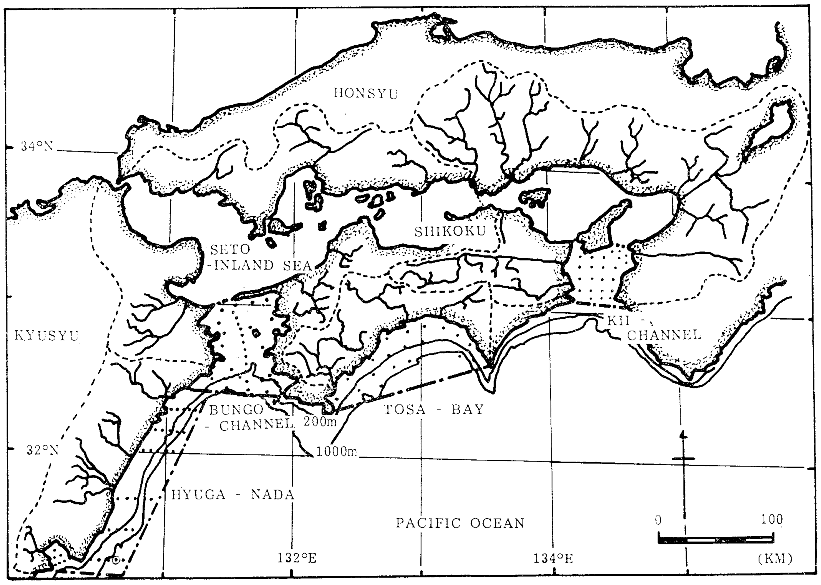

Fig. 1. Map of the Bungo Channel, Hyuga-Nada, Tosa Bay and the Kii Channel. Salinity observed stations are shown by black dottes. Major rivers are also shown. Thin full line shows the bottom contour and numbers the depth in meters. Broken line shows the drainage area.

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* Received August 10, 1985

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ever, the behavior of fresh water flown into the shelf sea has been little known.

This paper is concerned, by data analysis, with the frequency response characteristics of the variation of fresh water content in the shelf sea waters to that of river discharge from the land and backward inner coastal sea in the Bungo Channel, Hyuga-Nada, Tosa Bay and the Kii Channel in the south western part of Japan.

2. Data analysis

The Bungo Channel, Hyuga-Nada, Tosa Bay and the Kii Channel (Fig. 1) situated in the south western part of Japan are classified into two types of continental shelf sea. The Bungo Channel and the Kii Channel have inner coastal sea, Seto Inland Sea, at the back of them, whereas Hyuga-Nada and Tosa Bay abut on the land.

Salinities were observed every month from 1977 to 1981 at 0 m, 10 m, 20 m, 30 m and 50 m depths at 22 stations in the Bungo Channel, at 30 stations in Huga-Nada, at 20 stations in Tosa Bay and at 27 stations in the Kii Channel by Ehime, Ooita, Miyazaki, Kochi, Tokushima and Wakayama Prefecture Fisheries Observatories. Salinities from the sea surface to 50 m depth at each station were averaged by a depth weighting method. The fresh water content F in each area is estimated every month by the equation,

$$F=\sum_i\left(\frac{S_0-S_i}{S_0}\right)\times V_i\,. \qquad (2\text{-}1)$$

Fig. 2. Seasonal variation of representative salinity in the open ocean which is obtained by averaging salinity at 50 m depth at Stn. 4 off Toi in Hyuga-Nada shown by open circle in Fig. 1 from 1977 to 1981.

Here S_0 means the representative salinity in the open ocean, S_i the depth average salinity at the station i and V_i the water volume occupied by the station i. S_0 is made equal to the salinity at 50 m at Stn. 4 off Toi in Huga-Nada, which is shown by open circle in Fig. 1, because it usually showed the highest salinity in these areas. Since the salinity at 50 m at Stn. 4 shows a prominent seasonal variation, monthly values shown in Fig. 2 are used for S_0 in the following analysis.

The river discharge over the coastline is estimated by summing the mothly discharges of the first-class rivers, whose discharge are gauged every day, and those of small rivers. The discharge of small river is estimated by multiplying the discharge of neighboring first-class river by some factor. This factor is determined by the drainage area ratio of small river to the neighboring first-class river. The effects of direct precipitation on the shelf waters and evaporation from the sea surface of shelf wates are neglected, because they have nearly the same volume flux. Moreover the fresh water movements from one area to another area are neglected, because the most fresh water are moved away at the open end of each area by the strong Kuroshio which flows along the coast of Kyushu, Shikoku and Honshu Islands.

River discharge and fresh water content in each area are obtained every month from 1977 to 1981 and the average and standard deviation values are calculated (Table 1). Normalized monthly variations of river discharge and fresh water content with the average and standard deviation values in each area are shown in Fig. 3. River discharge has one prominent peak in a year and the local maximum fresh water

Table 1. Average and standard deviation values of river discharge and fresh water content in each area from 1977 to 1981.

	Fresh water discharge (km^3/month)		Fresh water content (km^3)	
	Ave.	S.D.	Ave.	S.D.
Bungo	2.50	1.71	4.07	2.48
Hyuga	1.51	1.24	2.58	2.91
Tosa	0.88	0.66	1.31	1.58
Kii	2.37	1.20	3.51	0.91

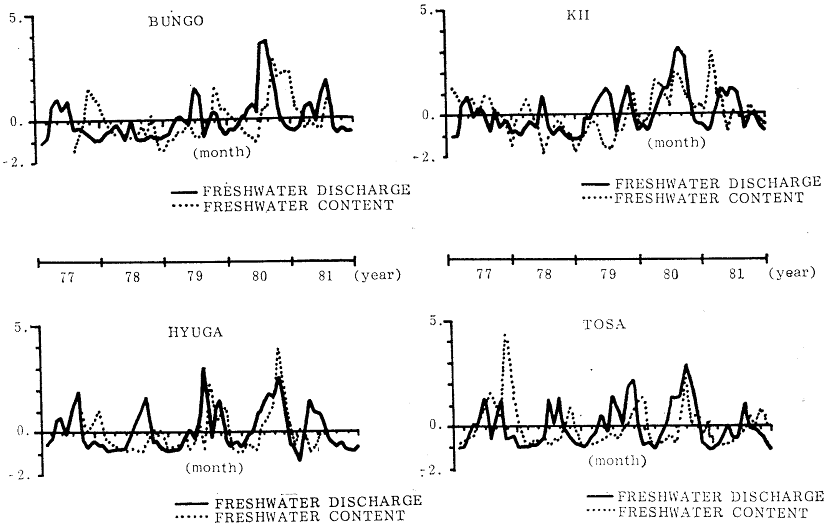

Fig. 3. Normalized monthly variations of river discharge and fresh water content from 1977 to 1981.

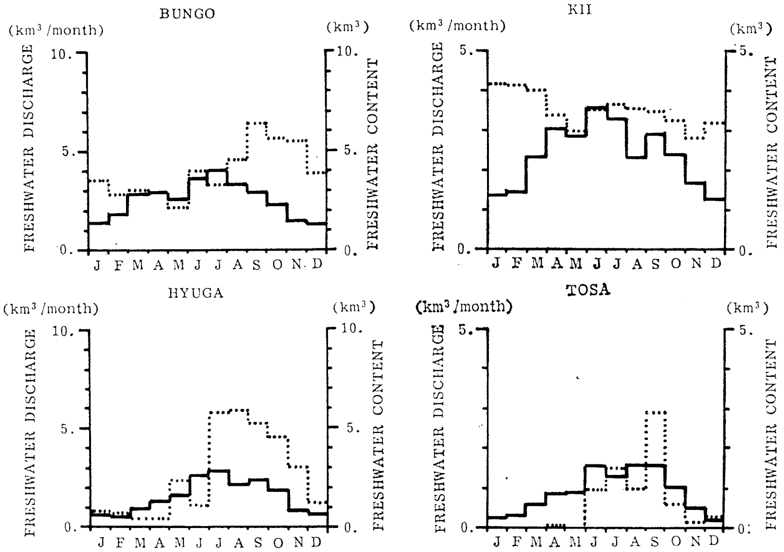

Fig. 4. Average seasonal variations of river discharge (full line) and fresh water content (broken line) from 1977 to 1981.

content occurs several months after that of river discharge. The average seasonal variations of river discharge and fresh water content are obtained by averaging monthly values from 1977 to 1981 and shown in Fig. 4. The maximum river discharges occur in June or July, the rainy season in Japan. On the other hand, the maximum fresh water contents occur in July or September in the Bungo Channel, Hyuga-Nada and Tosa Bay, and in January in the Kii Channel only.

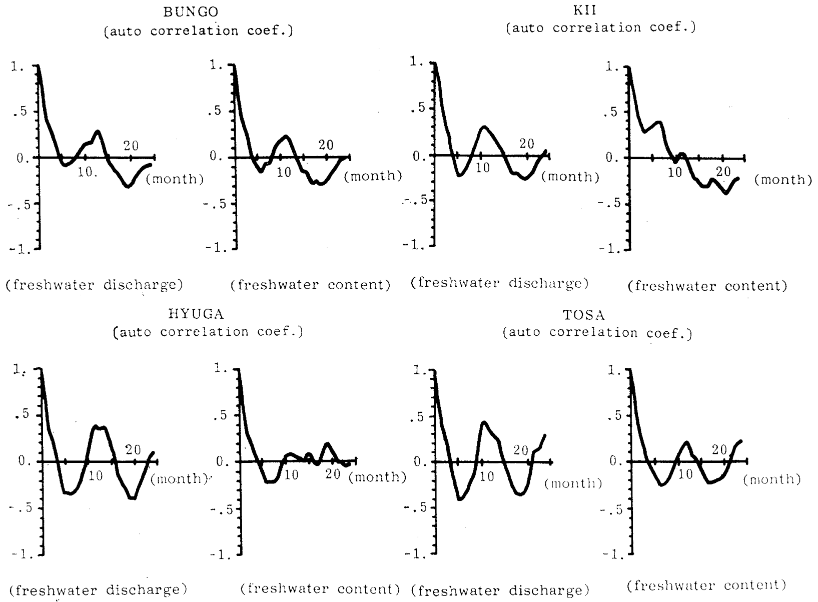

Fig. 5. Auto-correlation coefficients of river discharge and fresh water content variations.

3. Response function

We consider a linear system model which has one input (river discharge) and one output (fresh water content) in each area. The output signal is represented by the following convolution integral of the input signal,

$$y(t)=\int_0^\infty x(t-\tau)\cdot h(\tau)d\tau \qquad (3\text{-}1)$$

Here, $y(t)$ denotes the fresh water content, $x(t)$ the river discharge and $h(\tau)$ the unit response function of each area. The oceanic characteristics of each area is represented by the unit response function $h(\tau)$. There are four methods to estimate $h(\tau)$; (1) the direct method (*e.g.* HINO, 1977), (2) the correlation method (*e.g.* BOX and JENKINS, 1976), (3) the integral transfer method (*e.g.* FUJITA, 1982) and (4) the cross spectral method (*e.g.* HINO, 1977). Here the correlation method is used for estimating the unit response function for its high precision and simplicity.

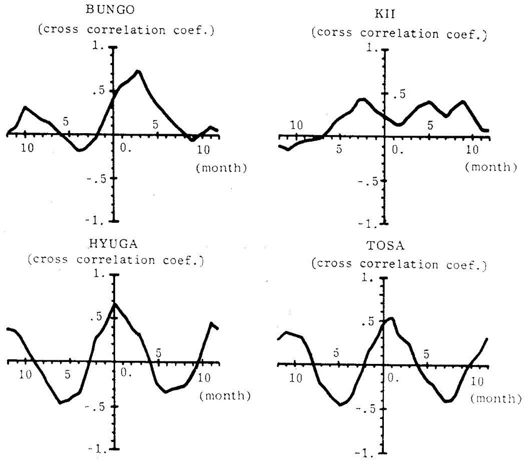

Fig. 6. Cross-correlation coefficient between river discharge and fresh water content variations.

At first the auto-correlation coefficients of variations of river discharge and fresh water content are calculated in each area as shown in Fig. 5. The seasonal variations seem to dominate except for the fresh water content in the Kii Channel. The cross-correlation coefficient between the variations of river discharge and fresh water content in each area is shown in Fig. 6. The correlation is not so high in the Kii Channel only. The unit response function is estimated by the correlation method as,

$$C_{xy}(\tau)=\int_0^T h(\eta)\cdot C_{xx}(\tau-\eta)\,d\eta. \qquad (3\text{-}2)$$

Here, $C_{xy}(\tau)$ denotes the cross-correlation coefficient between the variations of river discharge $x(t)$ and fresh water content $y(t)$, T the observation period and $C_{xx}(\tau)$ the auto-correlation coefficient of the variation of river discharge. Equation (3-2) in finite difference with unit time of one month is written in matrix form,

$$\begin{bmatrix} C_{xx}(0), C_{xx}(1), \cdots\cdots, C_{xx}(\mathrm{m}) \\ C_{xx}(1), \quad \cdot \quad \cdot \qquad , C_{xx}(\mathrm{m}-1) \\ \vdots \qquad \cdot \quad \cdot \qquad \vdots \\ \vdots \qquad\quad \cdot \quad \cdot \quad \vdots \\ C_{xx}(\mathrm{m}), C_{xx}(\mathrm{m}-1), \cdots C_{xx}(0) \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ \vdots \\ h(\mathrm{m}) \end{bmatrix} = \begin{bmatrix} C_{xy}(0) \\ C_{xy}(1) \\ \vdots \\ \vdots \\ C_{xy}(\mathrm{m}) \end{bmatrix}, \tag{3-3}$$

where m is taken as 7 months for the Bungo and Kii Channels, 6 months for Tosa Bay and 5 months for Hyuga-Nada from Fig. 5. The unit response function $h(\tau)$ is obtained by solving the linear simultaneous equations (3-3). The estimated discrete unit response function is shown in Fig. 7. The characteristics of frequency response of the variation of fresh water content to that of river discharge, which has the variability with wide range of frequency, is known by the Fourier transform of unit response function. However, it is very difficult to carry out the Fourier transform of the discrete unit response function. Therefore the discrete unit response function is approximated by a simple analytical function. The principle of approximation is as follows: In the Bungo Channel and the Kii Channel which have the inner coastal sea at the back of them, the unit response function is expressed by the summation of exponential functions and solution of diffusion equation as

$$h(\tau) = \sum_{n=1}^{2} a_n e^{-b_n \tau} + \frac{A}{\sqrt{\tau}} e^{-B/\tau} . \tag{3-4}$$

In Hyuga-Nada and Tosa Bay abutting on the land, the unit response function is expressed by the summation of exponential functions as

$$h(\tau) = \sum_{n=1}^{2} a_n e^{-b_n \tau} . \tag{3-5}$$

Coefficients a_n, b_n, A and B are obtained by the nonlinear least square method (Quasi Marquart method). The analytical forms of unit response function are obtained as

$$\left.\begin{array}{ll} \text{Bungo} & h(\tau) = 0.065\, e^{-0.247\tau} + 0.062\, e^{-0.246\tau} \\ & \qquad + \dfrac{0.211}{\sqrt{\tau}} e^{-0.610/\tau} , \\ \text{Kii} & h(\tau) = 3.378\, e^{-13.08\tau} - 3.049\, e^{-6.523\tau} \\ & \qquad + \dfrac{0.756}{\sqrt{\tau}} e^{-5.111/\tau} , \\ \text{Hyuga} & h(\tau) = 0.231\, e^{-0.298\tau} + 0.301\, e^{-0.408\tau} , \\ \text{Tosa} & h(\tau) = 0.600\, e^{-0.100\tau} - 0.349\, e^{-0.900\tau} . \end{array}\right\} \tag{3-6}$$

They are shown by broken lines in Fig. 7. The variation of fresh water content estimated by Eq. (3-1) with the analytical form of unit response function (3-6) and observed river discharge is shown in Fig. 8. The coincidence between the variation of estimated fresh water content and that of observed one is fairly good except for the Kii Channel.

The amplitude ratio and phase difference between the variation of river discharge and that of fresh water content are estimated by Fourier transform of the analytical form of unit response function;

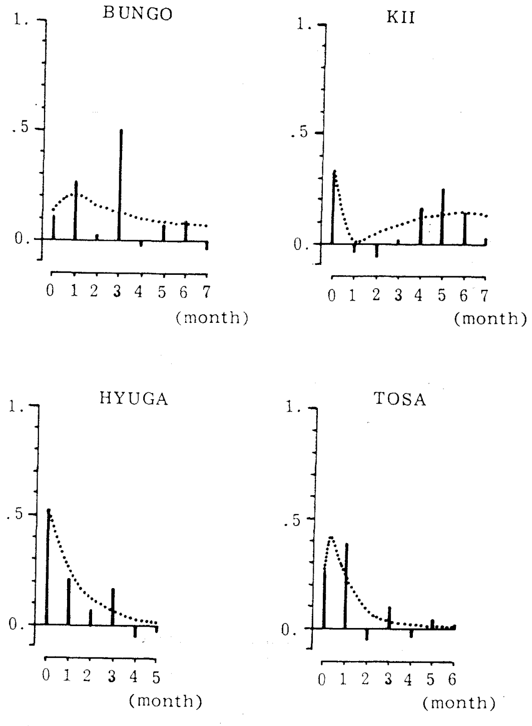

Fig. 7. Estimated unit response function. Broken line denotes the approximated analytical function to the discrete estimated one.

$$h(\tau)=\sum_n a_n e^{-b_n\tau}+\frac{A}{\sqrt{\tau}}e^{-B/\tau}$$

$$\begin{aligned}H(\omega)&=\int_0^\infty\left(\sum_n a_n e^{-b_n\tau}+\frac{A}{\sqrt{\tau}}e^{-B/\tau}\right)e^{-i\omega\tau}\,d\tau\\&=\int_0^\infty\sum_n a_n e^{-b_n\tau}\,e^{-i\omega\tau}\,d\tau\\&+A\int_0^\infty\frac{1}{\sqrt{\tau}}e^{-B/\tau}\,e^{-i\omega\tau}\,d\tau\\&=\sum_n\frac{a_nb_n}{\omega^2+b_n^2}-i\sum_n\frac{\omega a_n}{\omega^2+b_n^2}+\frac{A}{\sqrt{\omega}}e^{-\sqrt{2B\omega}}\\&\qquad(\cos\sqrt{2B\omega}-\sin\sqrt{2B\omega}),\end{aligned}\qquad(3\text{-}7)$$

where $H(\omega)$ denotes the frequency response function. Therefore, the amplitude ratio $|H(\omega)|$ and phase difference $\phi(\omega)$ are

$$|H(\omega)|=\sqrt{H_R^2(\omega)+H_I^2(\omega)},\qquad(3\text{-}8)$$

$$\phi(\omega)=\tan^{-1}\left(\frac{H_I(\omega)}{H_R(\omega)}\right),\qquad(3\text{-}9)$$

$$\left.\begin{aligned}H_R(\omega)&=\sum_n\frac{a_nb_n}{\omega^2+b_n^2}+\frac{A}{\sqrt{\omega}}e^{-\sqrt{2B\omega}}\\&\qquad(\cos\sqrt{2B\omega}-\sin\sqrt{2B\omega}),\\H_I(\omega)&=\sum_n\frac{\omega a_n}{\omega^2+b_n^2}.\end{aligned}\right\}\qquad(3\text{-}10)$$

The estimated amplitude ratio and phase difference in each area are shown in Fig. 9. As for the prominent seasonal variation, the Bungo Channel has the largest amplitude ratio of about 0.9 month; the amplitude of seasonal variation of fresh water content in the Bungo Ceannel is obtained by multiplying the amplitude of river discharge to the Bungo Channel by 0.9 month. The amplitude ratios of seasonal variation in Hyuga-Nada, Tosa Bay and the Kii Channel are 0.8 month, 0.5 month and 0.5 month, respectively. The phase difference of seasonal variation in the Bungo Channel, Hyuga-Nada, Tosa Bay and the Kii Channel are 0.5 month, 0.3 month, 0.3 month and 6 months, respectively. These values will be useful for comparing these areas from the viewpoint of the resistivity of shelf waters against the pollution and of the productivity of shelf waters.

4. Discussion

The difference between full line and broken line in the amplitude ratio and the difference between full line and 90° in the phase difference in Fig. 9 show the openness of each area, because the broken line and 90° represent the amplitude ratio and the phase difference of a closed basin, respectively. The opennes of each area relates to the water exchange ability, which depends on the flow system in each area. For example, the tidal currents are dominant in the

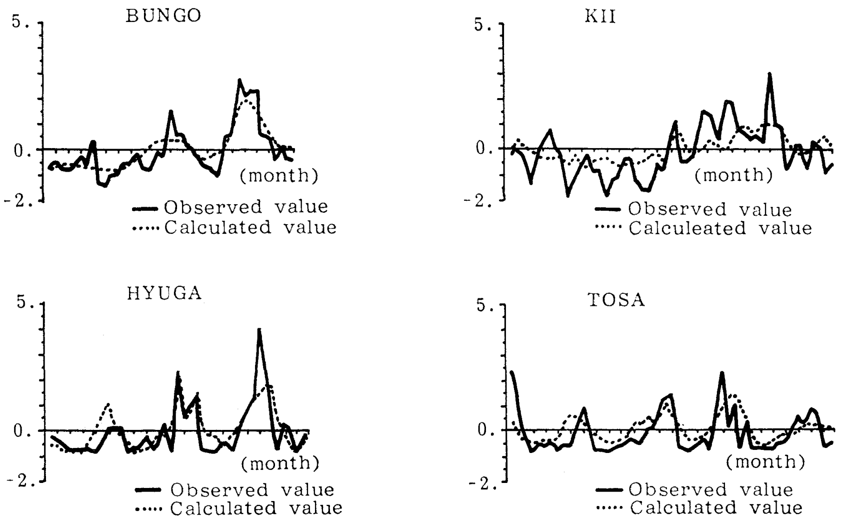

Fig. 8. Observed fresh water content (full line) and estimated one (broken line) with the approximated unit response function.

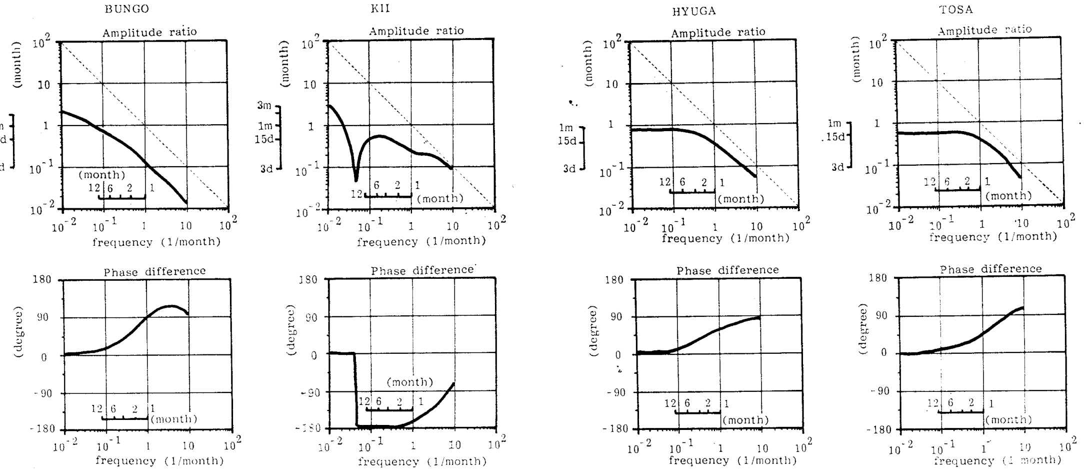

Fig. 9. Amplitude ratio of the variation of fresh water content to that of river discharge (upper) and phase difference of the variation of fresh water content to that of river discharge (lower).

Bungo and the Kii Channels because of their shallowness and of presence of the inner coastal sea at the back of them. On the other hand, tidal currents are not dominant, but flows of other types such as frontal eddies are dominant in Hyuga-Nada and Tosa Bay. The Bungo Channel has the most closed character among the four areas for the input signal with one-year period but Hyuga-Nada and Tosa Bay have more closed character than the Bungo Channel for the input signal with several-month period. The tidal current works as a strong stirrer for the fresh water input with several-month period but does not for the fresh water input with one-year period.

Although the Kii Channel shows the peculiar characteristics of amplitude ratio and phase difference, the cross-correlation coefficient is low as shown in Fig. 5, so that the reliability of frequency response function of the Kii Channel is fairly low. The salinity observation area of the Kii Channel (Fig. 1) is narrow, because the Kii Channel has three large rivers along its coastline while the Bungo Channel has no large river along its coastline. More salinity data are needed in wider area and in longer period for estimating the accurate frequency response function of the Kii Channel.

There are many elementary physical processes in shelf waters such as tidal current, density-driven current, wind-driven current. They are influenced by the irregularities of horizontal geometry and bottom topography of shelf area. Some standard measures are needed for comparing characteristics of continental shelf area governed by different composite physical processes. In this context, the frequency response function is one of useful measures. Moreover, it will be able to predict the time variation of concentration of any man-made contamination from land in shelf waters if the frequency response function is correctly given.

Acknowledgements

The authors express their sincere thanks to Dr. H. TAKEOKA of Ehime University for his helpful discussions. The data processing was carried out on a FACOM M-180 II AD of the Data Processing Center of Ehime University. This study is a part of the special project research "The ocean characteristics and their change" by the Ministry of Education, Science and Culture, Japan.

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陸棚水中の淡水の周波数応答特性

柳 哲雄・大庭哲哉

要旨: 陸棚水中の淡水存在量の変動の河川流出量の変動に対する周波数応答特性を日向灘，豊後水道，土佐湾，紀伊水道での資料解析により明らかにした。河川流出量の変動に卓越する1年周期に対して，陸棚水中の淡水存在量の振幅はこれらの海域で0.5カ月から0.9カ月，変動の位相差は0.3月から6ケ月と異なることがわかった。