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A non-dissipative internal bore*

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Abstract: An analytical solution of an internal bore is derived using a two-layer fluid model. This bore differs from the classical one in that energy dissipation does not occur as it proceeds.

1. Introduction

Internal bores in the ocean and lakes which are often accompanied by large-amplitude oscillations, have been reported by numerous authors; notably HALPERN (1971), THORPE (1971), HUNKINS and FLIEGEL (1973), WINANT (1974), IVANOV and KONYAEV (1976), and FARMER (1978), among others. These bores may be caused by tidal currents, transformation of a long internal wave, slackening of wind over narrow lakes, or intrusion of surface or bottom water from the open sea, although the mechanism of their formation is not fully understood. The similar phenomenon in the atmosphere was reported by SMITH *et al.* (1986).

Most of the theoretical models so far studied to describe the bores have been based on a KdV equation or a modified KdV equation (e. g. LEE and BERADSLEY, 1974). However, the applicability of the KdV equation is rather doubtful because the amplitude of the phenomenon is often so large that the assumption of weak non-linearity can no longer be valid.

The purpose of this note is to show that, using a simple two-layer model, a solution of a large amplitude internal bore can be derived.

2. Equations of motion and bore solution

Consider irrotational two-dimensional motion in a two-fluid system, in which a layer of lighter fluid overlies a layer of heavier fluid, resting on a horizontal impermeable bed. For simplicity the free surface is replaced with a rigid lid, eliminating the surface wave. Both lower and upper fluids are incompressible with homogeneous densities ρ_1 and ρ_2. The depth of the lower layer at infinity h1 is assumed to be smaller than that of the upper layer h_2.

Then assuming a steady state solution with a uniform flow of constant velocity c at infinity, and using the three conservation laws of mass, enrgy and flow force, we can obtain the following ordinary differential equation (see MIYATA, 1985).

$$\frac{M}{2}\left(\frac{d\zeta}{d\xi}\right)^2+K=0, \tag{1}$$

Where ζ represents the nondimensional displacement (normalized by h_1) of the interface from the undisturbed position, ξ is the horizontal coordinate defined by $h_1\xi = x - ct$, c being the velocity of the stationary bore.

$$M=\frac{2F^2r^2(1+rs)}{3(r+s)},\quad K=\frac{-\zeta^4+D\zeta^3+E\zeta^2}{B\zeta+1},$$

$$B=\frac{r^2s-1}{r(1+rs)},\quad D=\frac{r^2-s+r(F^2-1)(1-s)}{r+s},$$

$$E=-r(F^2-1),$$

$$r=\frac{h_2}{h_1},\quad s=\frac{\rho_2}{\rho_1},\quad F=\frac{c}{c_0},\quad c_0=\sqrt{\frac{gh_1(1-s)r}{1+rs}}.$$

Now consider the special case when

$$F=\frac{\sqrt{(1+r)(r+s)}}{\sqrt{r}\,(1+\sqrt{s})}. \tag{2}$$

Then Eq.(1) can be simplified to:

$$\frac{d\zeta}{d\xi}=\frac{\zeta(2\zeta-D)}{\sqrt{2M(B\zeta+1)}}. \tag{3}$$

Upon integration we can obtain the solution of this equation, that is,

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$$\xi=\frac{\sqrt{r(1+r)(1+rs)}}{\sqrt{3}\,(r-\sqrt{s})}\left(ln\frac{\sqrt{B\zeta+1}+1}{\sqrt{B\zeta+1}-1}\right.$$

$$\left.+m\,ln\frac{m-\sqrt{B\zeta+1}}{m+\sqrt{B\zeta+1}}\right), \qquad (4)$$

where

$$m=\sqrt{\frac{\sqrt{s}\,(1+r)(1+r^2\sqrt{s})}{r(1+\sqrt{s})(1+rs)}}, \quad 1\leq\sqrt{B\zeta+1}\leq m.$$

3. Discussion

The solution (4) describes a shock wave or an internal bore advancing into still water. However, this bore has no energy dissipation as opposed to the classical concept of bores where energy is always dissipated along the shock line. Such non-dissipative bores of small amplitude were discussed by KAKUTANI and YAMASAKI (1978), and FUNAKOSHI (1985). Thus the bore obtained above can be regarded as a large amplitude generalization of their results.

Since the height of the bore is determined from the equation $m=\sqrt{B\xi+1}$, the non-dimensional amplitude A is given by the following:

$$A=\frac{r-\sqrt{s}}{1+\sqrt{s}}. \qquad (5)$$

It should be noted that once the environmental parameters r and s are given, the amplitude is determined. From (2) the non-dimensional veleocity F, which also depends or r and s only, is always greater than 1.

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非消散内部ボア

宮　田　元　靖

要旨：海洋内部におけるボアの解を簡単な２層モデルを使って求めた。このボアは従来のボアと異なり進行中にエネルギーの消散を伴なわない。

APPENDIX: ERRATA for the previous two papers by the author

I：“An internal solitary wave of large amplitude”, La mer **23**, 1985.

Page	Misprints	Should read
p. 44. Eq. (11)	$i^3(z-h)_\varepsilon$	$i^3(z-h)^3$
p. 46. Line 1	$\sqrt{\frac{2}{M}}\,d\xi$	$\pm\int\sqrt{\frac{2}{M}}\,d\xi$
p. 46. Lines 1, 2, & 3	$E+D\zeta-\zeta^2$	$\zeta^2-D\zeta-E$
p. 48. Eq. (A1)	$=$ Asech...	$\eta=$ Asech...
p. 48. Eq. below (A2)	$g(\rho_2-\rho_2)$	$g(\rho_1-\rho_2)$

II：“Comment on Boussinesq's long wave equation”, La mer **24**, 1986.

Page	Misprints	Should read
p. 60. Eq. (7)	$\frac{\partial\phi}{\partial t}$	$\frac{\partial\phi}{\partial x}$
p. 60. Eq. (12)	$\frac{1}{3}\frac{\partial\eta}{\partial t}$	$\frac{1}{\varepsilon}\frac{\partial\eta}{\partial t}$
p. 60. Eq. (13)	$\frac{1}{3}\frac{\partial\eta}{\partial x}$	$\frac{1}{\varepsilon}\frac{\partial\eta}{\partial x}$
p. 60. Eq. (18)	$\frac{\partial^2 U}{6x^2}$	$\frac{\partial^2 U}{\partial x^2}$
p. 60. Eq. (20)	$\left(\frac{\partial\eta}{\partial x}\right)^{1-}$	$\left(\frac{\partial\eta}{\partial x}\right)^{-1}$
p. 61. Eq. (22)	$-c\frac{d}{d\xi}$	$-c\frac{d\eta}{d\xi}$
p. 61. Eq. (23)	$\varepsilon U=\frac{B}{\eta}+C$	$\varepsilon U=\frac{B}{\eta}+c$
p. 61. Eq. (28)	$B=-C$	$B=-c$
p. 61. Eq. (28)	$D=1+\frac{C^2}{2}$	$D=1+\frac{c^2}{2}$
p. 61. Eq. (28)	$E=\frac{1}{2}-C^2$	$E=-\frac{1}{2}-c^2$
p. 61. Eq. (30)	$\frac{3\varepsilon}{1+\delta}$	$\frac{3\varepsilon}{1+\varepsilon}$
p. 61. Eq. (34)	$\varepsilon U\frac{\partial U}{\partial t}+\frac{\partial\zeta}{\partial t}$	$\varepsilon U\frac{\partial U}{\partial x}+\frac{\partial\zeta}{\partial x}$